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# RAN-2503000505033001

## T.Y.B.Sc. (Sem. - V) Examination October - 2025

## Mathematics Major (Paper - MH-MJ-503) Real Analysis - I

**Time: 2 Hours ]** **[ Total Marks: 25**

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| **સૂચના : / Instructions**<br>(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.<br>Fill up strictly the above details on your answer book<br>(2) All questions are compulsory.<br>(3) Figures to the right indicate marks of the corresponding question. | Seat No.:<br><br>Student's Signature |

**Q.l. Answer the following (Any Five):** **05**

1. If f $(x) = 1 + sinx$ $(-\infty < x < \infty)$, $g(x) = x^2$ $(0 \le x < \infty)$ then find $gof$.
2. Define Characteristic function.
3. Is a sequence $\{n\}_{n=1}^{\infty}$ convergent? Justify your answer.
4. Write first six terms of Fibonacci sequence.
5. Find least upper bound of the set $A = \{x \in R \,/\, x^2 \ge \pi\}$
6. If A = $\{x \in N/2 \le x \le 6\}$ and $B = \{x \in N/x < 9$ *and x is an even number*$\}$, then obtain A×B.

**Q.2. Answer the following (Any Two):** **10**

1. Define Countable set. Also prove that if $f$: $A \to B$, and the range of f is uncountable then the domain of f is uncountable.
2. Let $f(x) = tanx$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then
   (i) Find domain of $f$ (ii) Find range of $f$.
   (iii) If $A = \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]$, $B = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ then $f(A \cap B) = f(A) \cap f(B)$?
3. Let $f : Z \to E$, where E denote the set of all even integers, f (n) = 2n, check whether f is one-one and onto or not.

**Q.3. Answer the following (Any Two):** **10**

1. If the sequence of real numbers $\{Sn\}_{n=1}^{\infty}$ is converges to L, then prove that $\{Sn\}_{n=1}^{\infty}$ cannot converges to a limit distinct from $L$.
2. If $S = \{Sn\}_{n=1}^{\infty} = \{2n - 1\}_{n=1}^{\infty}$ and $N = \{n_i\}_{i=1}^{\infty} = \{i^2\}_{i=1}^{\infty}$ then find $S_5$, $S_9$, $n_2$, $S_{n_4}$.
3. If $S = \{Sn\}_{n=1}^{\infty}$ is a sequence of real numbers and if $\lim_{m\to\infty} S_{2m} = L$, $\lim_{m\to\infty} S_{2m-1} = L$ then prove that $S_n \to L$ as $n \to \infty$.